





...

:



k



. l<sub>0</sub>

k

m

$\vec{P}$

: ✓

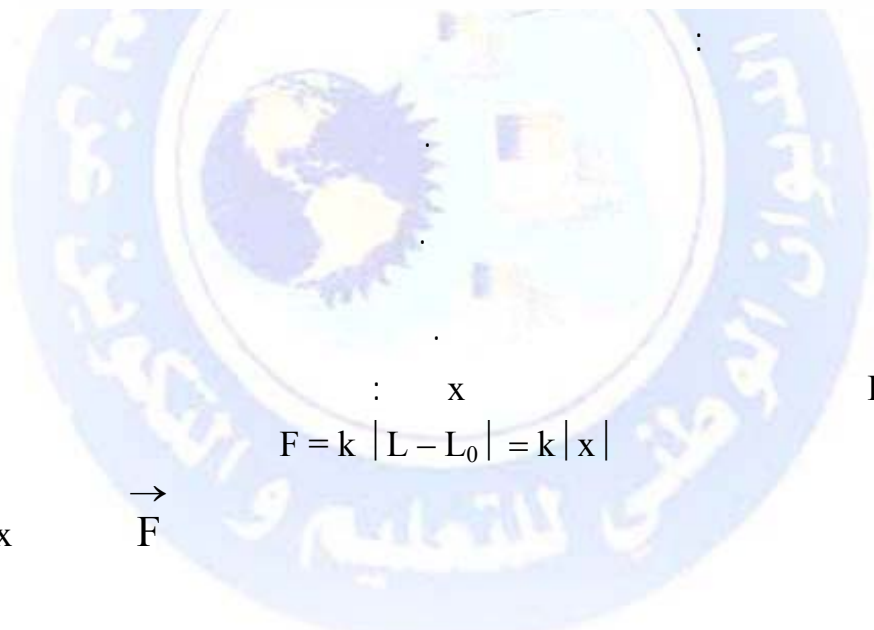
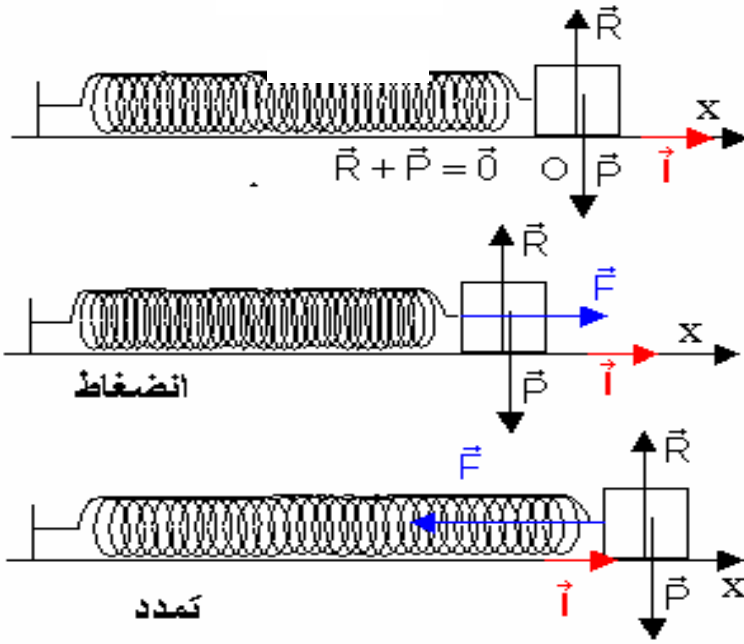
$\vec{R}$

✓

l  
(O,  $\vec{i}$ ,  $\vec{j}$ )

. m

:



- $\vec{P}$
- $\vec{R}$
- $\vec{F}$

$$F = k |L - L_0| = k |x|$$

$\vec{F}$ ,  $x$

$$\vec{F} = k \cdot x \cdot \vec{i} : x < 0$$

$\vec{F}$

$$\vec{F} = -k \cdot x \cdot \vec{i} : x > 0$$

( )

( )



( - )

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■

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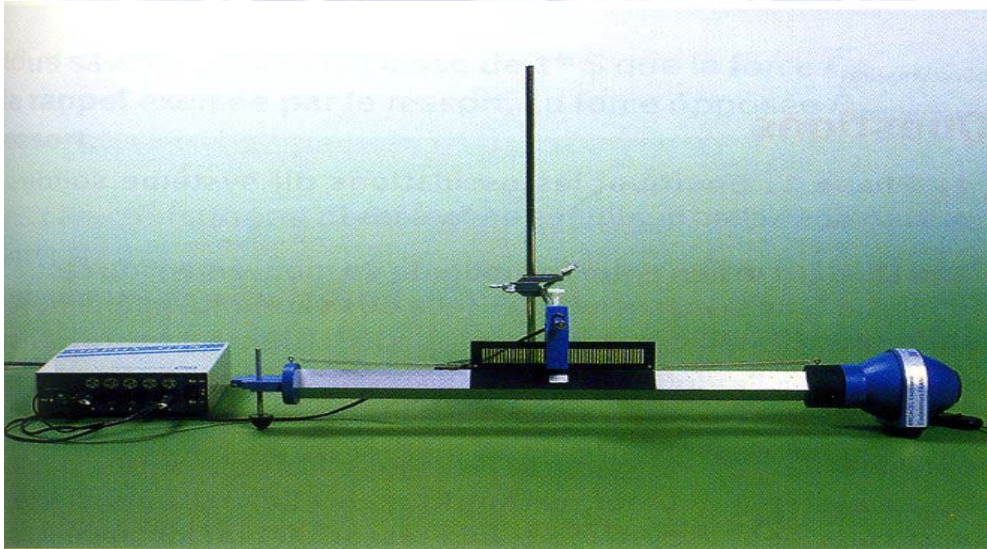
■

■

:

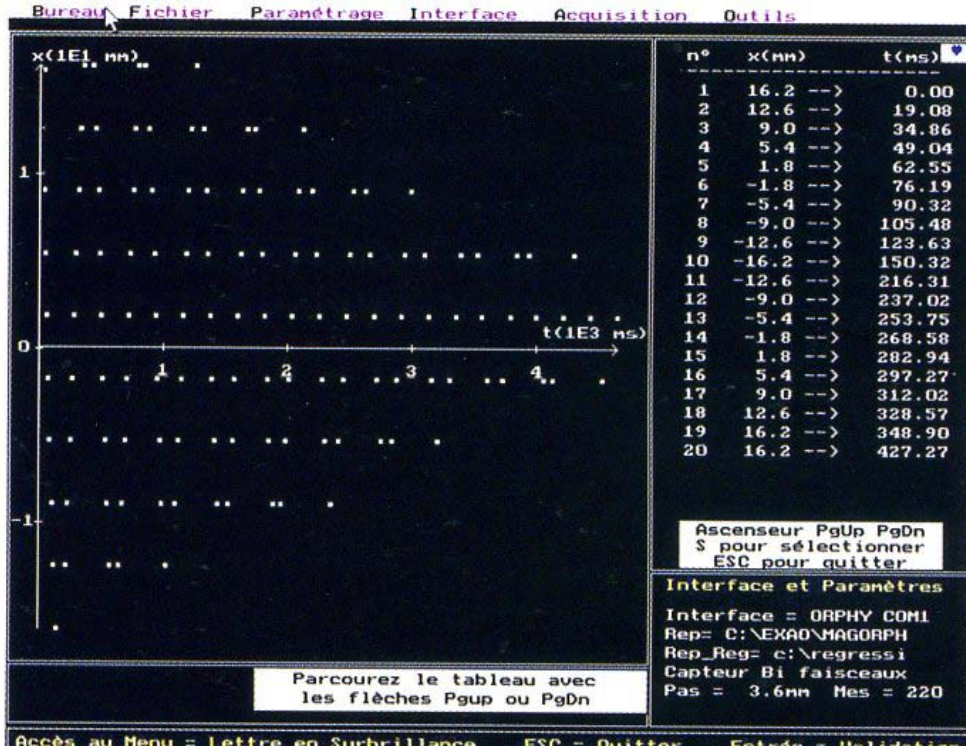
$m = 80 \text{ g}$  S

$k_1 = 10 \text{ N/m}$  ( )



M

$k = 2 k_1$



-1

-2

$$T_0 = 2\pi \sqrt{\frac{m}{k}}$$

-3

-4

:

-1

.( )

: - -2

$$.T = 0,4 \text{ s}$$

- :

$$T_0 = 2\pi \sqrt{\frac{m}{k}}$$

:

$$T_0 = 2\pi \sqrt{\frac{80 \times 10^{-3}}{20}} = 0,397 \text{ s}$$

$$T_0 = 0,4 \text{ s}$$

$$T \approx T_0$$

-3

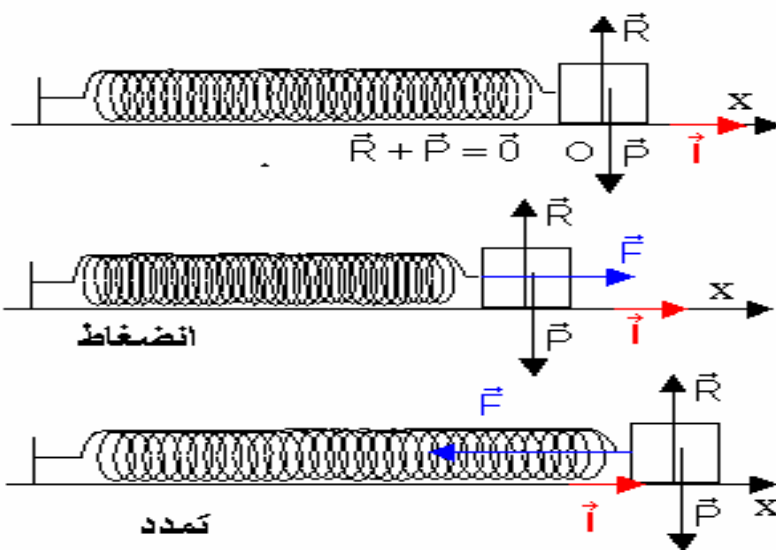
-4

$$T_0 = 2\pi \sqrt{\frac{m}{k}}$$

:

( )

(O,  $\vec{i}$ ,  $\vec{j}$ )



$$\vec{P} + \vec{R} + \vec{F} = m \vec{a}_G$$

$$= : \vec{a}_G$$

$$0 + 0 - kx = m a_x$$

$$m a_x + kx = 0$$

$$a_x = \frac{d^2 x}{dt^2}$$

$$\frac{d^2 x}{dt^2} + \frac{k}{m} \cdot x = 0$$

$$\frac{d^2 x}{dt^2} + \frac{k}{m} \cdot x = 0$$

$$x = x_m \cdot \cos \left( \frac{2\pi \cdot t}{T_0} + \varphi_0 \right)$$

(m)

=  $x_m$

(s)

=  $T_0$



$$x(t) = x(t + T_0)$$

$$x_m \cdot \cos\left(\frac{2\pi \cdot t}{T_0} + \varphi_0\right) = x_m \cdot \cos\left(\frac{2\pi \cdot t}{T_0} + 2\pi + \varphi_0\right)$$

$$\left(\frac{2\pi \cdot t}{T_0} + \varphi_0\right) = \left(\frac{2\pi \cdot t}{T_0} + 2\pi + \varphi_0\right)$$

$$t=0 \quad \left(\frac{2\pi \cdot t}{T_0} + \varphi_0\right) = \varphi_0$$

(x) :

$$\frac{dx}{dt} = -x_m \cdot \frac{2\pi}{T_0} \cdot \sin\left(\frac{2\pi \cdot t}{T_0} + \varphi_0\right)$$

( ) :

$$\frac{d^2x}{dt^2} = -x_m \cdot \frac{4\pi^2}{T_0^2} \cdot \cos\left(\frac{2\pi \cdot t}{T_0} + \varphi_0\right)$$

$$\frac{d^2x}{dt^2} + \frac{k}{m} \cdot x$$

$$\frac{d^2x}{dt^2} + \frac{k}{m} \cdot x = -x_m \cdot \frac{4\pi^2}{T_0^2} \cos\left(\frac{2\pi \cdot t}{T_0} + \varphi_0\right) + \frac{k}{m} x_m \cdot \cos\left(\frac{2\pi \cdot t}{T_0} + \varphi_0\right)$$

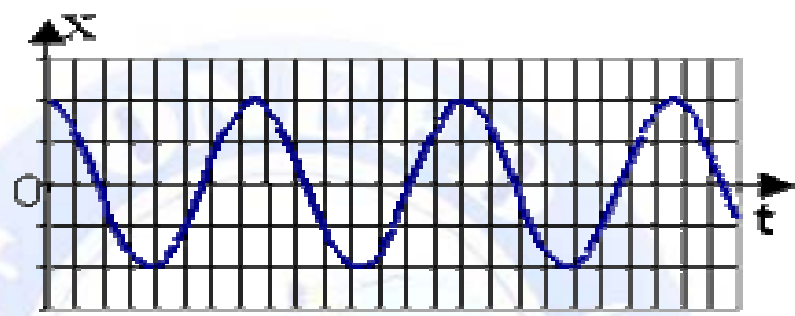
$$= \left(\frac{k}{m} - \frac{4\pi^2}{T_0^2}\right) \cdot x_m \cos\left(\frac{2\pi \cdot t}{T_0} + \varphi_0\right)$$

$$\frac{k}{m} - \frac{4\pi^2}{T_0^2} = 0 : \quad \frac{d^2x}{dt^2} + \frac{k}{m} \cdot x = 0$$

$$T_0 = 2\pi \cdot \sqrt{\frac{m}{k}}$$

:

$$f = \frac{1}{T_0} :$$



:  $\varphi$   $x_m$

.  $t = 0$

$$\frac{dx}{dt}(t = 0) = 0 \quad x(t = 0) = x_m > 0$$

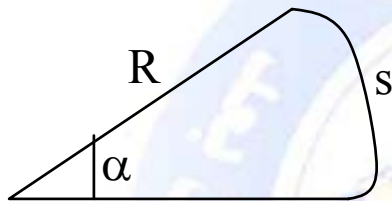
$$\frac{dx}{dt}(t = 0) = -x_m \cdot \frac{2\pi}{T_0} \cdot \sin(\varphi_0) = 0$$

:  $\sin \varphi_0 = 0$  :

$\varphi = \pi$  أو  $\varphi = 0$

$$x(t) = x_m \cdot \cos\left(\frac{2\pi \cdot t}{T_0}\right)$$

			:		
	G			G	✓
				[G]	
		[G] = M		G	:
		.G		[G] = M	✓
1	[G] = 1	G			✓



(rad)

$$[\alpha] = \frac{[s]}{[R]} = \frac{L}{L} = 1$$

			:		
				$\alpha$	
				$\alpha = \frac{s}{R}$	
				$[\alpha] = \frac{[s]}{[R]} = \frac{L}{L} = 1$	
				$\alpha$	✓
					✓
			:		
					✓
		[B].[A] = [AB] :			✓
		= n		[A] <sup>n</sup> A <sup>n</sup>	✓
u	e <sup>u</sup> , log(u), ln(u), tan(u), cos(u), sin(u) :				✓
				G	✓

$$[G] = L^a M^b T^c I^d J^e \theta^f N^g$$

:

	L	(m)
	M	(kg)
	T	(s)
	I	(A)
	$\theta$	(K)
	N	(mol)
	J	(Cd)

. k m T

$$. T_0 = f(k, m)$$

:

$$T_0 = C.k^a .m^b$$

. = C

: (F = k.x)

$$[k] = [F][L]^{-1}$$

$$[k] = M . T^{-2} \quad ; \quad [F] = M . L . T^{-2} \quad ;$$

$$[T_0] = [k]^a . [M]^b = M^a . T^{-2a} . M^b$$

$$[T_0] = M^{a+b} . T^{-2a}$$

$$: \quad [T] = T$$

$$\begin{cases} a + b = 0 \\ -2a = 0 \end{cases}$$

$$\begin{cases} a = -\frac{1}{2} \\ b = +\frac{1}{2} \end{cases}$$

$$T_0 = C \sqrt{\frac{m}{k}} :$$

$$C = 2\pi$$

C

:

:

C

:

k =

10 N/m

$$T = \frac{t}{10}$$

M(g)	140	160	180	200
t (s)	7,4	7,9	8,4	8,9
T(s)				
T <sup>2</sup> (s <sup>2</sup> )				

/1

$$T^2 = f(m)$$

/2

/3

/4

/5

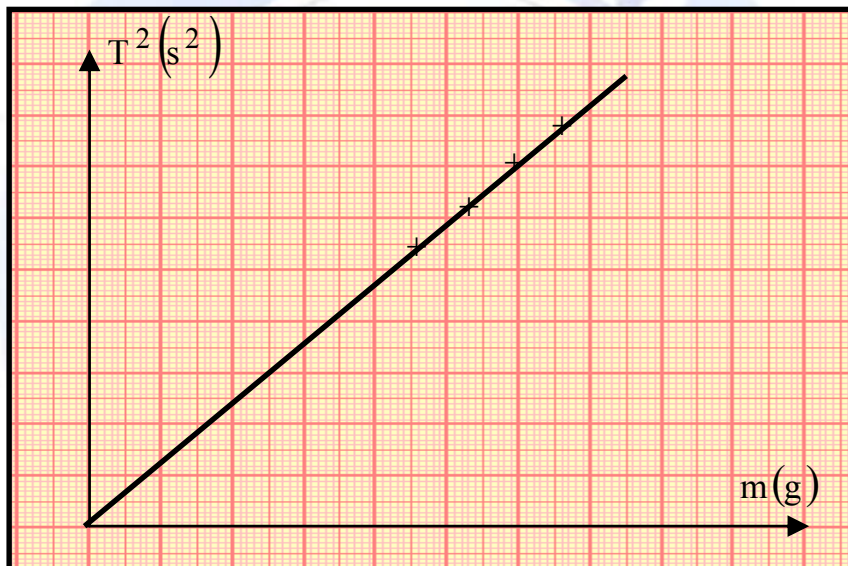
:

: /1

$$T = \frac{t}{10}$$

T(s)	0,74	0,79	0,84	0,89
T <sup>2</sup> (s)	0,548	0,624	0,705	0,792

: T<sup>2</sup> = f(m) /2



: /3

$$a = \frac{\Delta T^2}{\Delta m} = \frac{0,355 - 0}{90 \cdot 10^{-3} - 0} = 3,94 \text{ s}^2 / \text{kg}$$

$$a = 3,94 \text{ s}^2 / \text{kg}$$

:a.k /4

$$a.k = 3,94 \times 10 = 39,4$$

$$(1) \dots a = \frac{4\pi^2}{k} \quad a.k = 4.\pi^2 \quad 39,4 \simeq 4.\pi^2 \quad /5$$

<http://www.onefd.edu.dz>

$$(2) \dots T^2 = a.m :$$

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$$T = 2\pi \sqrt{\frac{m}{k}} : \quad T^2 = \frac{4\pi^2}{k} \cdot m : \quad (2) \quad (1)$$

:

. (s) T

.(kg) m

.(N / m) k

:

( " - " ) E<sub>c</sub>

.( ) E<sub>p e</sub>

$$E_M = E_c + E_{p e} :$$

:

$$E_c \quad (x = 0)$$

E<sub>p e</sub>

E<sub>p e</sub>

E<sub>c</sub>

$$x = x_m \cdot \cos \left( \frac{2\pi \cdot t}{T_0} + \varphi_0 \right)$$

:

$$V = \frac{dx}{dt} = -x_m \cdot \frac{2\pi}{T_0} \cdot \sin \left( \frac{2\pi \cdot t}{T_0} + \varphi_0 \right)$$

$$E_M = \frac{1}{2} m \cdot V^2 + \frac{1}{2} k \cdot x^2$$

$$E_M = \frac{1}{2} m \cdot x_m^2 \cdot \frac{4\pi^2}{T_0^2} \cdot \sin^2 \left( \frac{2\pi}{T_0} \cdot t + \varphi_0 \right) + \frac{1}{2} k \cdot x_m^2 \cdot \cos^2 \left( \frac{2\pi}{T_0} \cdot t + \varphi_0 \right)$$

:

$$T_0 = 2\pi \cdot \sqrt{\frac{m}{k}}$$

:

$$\frac{4\pi^2}{T_0^2} = \frac{k}{m} \quad T_0^2 = 4\pi^2 \cdot \frac{m}{k}$$

$$E_M = \frac{1}{2} m \cdot x_m^2 \cdot \frac{k}{m} \cdot \sin^2 \left( \frac{2\pi}{T_0} \cdot t + \varphi_0 \right) + \frac{1}{2} k \cdot x_m^2 \cdot \cos^2 \left( \frac{2\pi}{T_0} \cdot t + \varphi_0 \right)$$

$$E_M = \frac{1}{2} k \cdot x_m^2 \left[ \sin^2 \left( \frac{2\pi}{T_0} \cdot t + \varphi_0 \right) + \cos^2 \left( \frac{2\pi}{T_0} \cdot t + \varphi_0 \right) \right] = \frac{1}{2} k \cdot x_m^2 = C^{te}$$

$$E_M = \frac{1}{2} k \cdot x_m^2$$

:

:	(	-	)		$E_M = \frac{1}{2} k \cdot x_m^2$
---	---	---	---	--	-----------------------------------

(Microméga Hatier)



**Pendule élastique**

Ouvrir Enregistrer Copier Imprimer Calculatrice Aide Fermer

$t = 2.28$  s  
 $x = -0.03$  m  
 $\dot{x} = -1.86$  m · s<sup>-1</sup>  
 $\ddot{x} = 0.10$  m · s<sup>-2</sup>  
 $E_c = 250.00$  J  
 $E_{pe} = 0.00$  J  
 $E_m = 250.00$  J

**Animation :**

⏪ ⏩ ⏸ ⏹

**Exercices**

### Pendule élastique

Cette application permet de simuler le mouvement d'un pendule élastique dans un fluide sans vitesse initiale.

Vous pouvez modifier les paramètres :

- la masse du solide,
- la raideur du ressort,
- le coefficient de frottement fluide.

Pour déplacer le solide, vous pouvez saisir sa position initiale (onglet Paramètres) ou utiliser la souris sur l'animation.

Vous pouvez activer ou arrêter l'animation ou retrouver la position initiale

Paramètres Vecteurs Affichage **Enregistrement**

Durée de l'enregistrement 2 s

t (s)	$\ddot{x}$ (m · s <sup>-2</sup> )	$E_c$ (J)	$E_{pe}$ (J)	$E_m$ (J)
0	34.5	0	0.25	0.25
0.04	25.4	0.114	0.136	0.25
0.08	2.94	0.248	0.00181	0.25
0.12	-21.1	0.157	0.0934	0.25



- .1
- .2
- .3

$$E_M = \frac{1}{2} m \cdot V^2 + \frac{1}{2} k \cdot x^2 = C^{te}$$

$$\frac{dV}{dt} = x'' \quad V = \frac{dx}{dt} = x'$$

$$\frac{1}{2} m \cdot 2 \cdot V \cdot \frac{dV}{dt} + \frac{1}{2} k \cdot 2 \cdot x \cdot \frac{dx}{dt} = 0$$

$$m \cdot x' \cdot x'' + k \cdot x \cdot x' = 0$$

$$m \cdot x'' + k \cdot x = 0$$

$$x'' + \frac{k}{m} \cdot x = 0$$

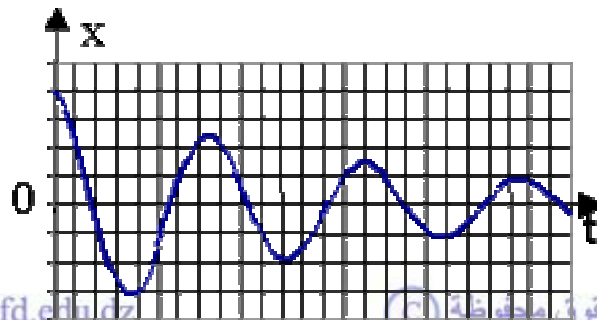
$$x'' + \frac{k}{m} \cdot x = 0$$

( )

T

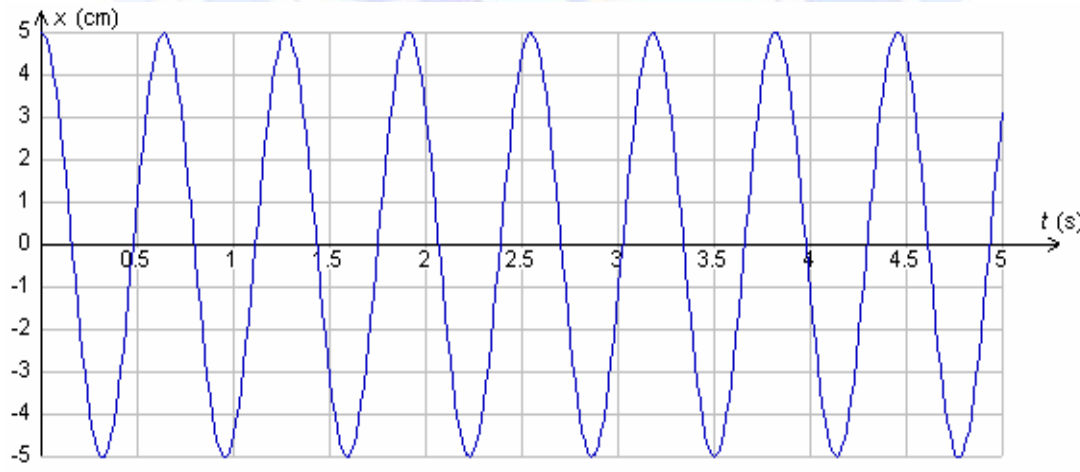
. T<sub>0</sub>

$$T \approx T_0 = 2\pi \cdot \sqrt{\frac{m}{k}}$$



:( )  
(Microméga Hatier)

Paramètres	Vecteurs	Affichage	Enregistrement
<b>Position initiale</b>		$x_0$ (-10 à 10) = <input type="text" value="5"/> cm	
<b>Masse</b>		$m$ (10 à 5 000) = <input type="text" value="2000"/> g	
<b>Raideur du ressort</b>		$k$ (1 à 1 000) = <input type="text" value="19,5"/> N · m <sup>-1</sup>	
<b>Coefficient de frottement</b>		$h$ (0 à 1 000) = <input type="text" value="0"/> N · s · m <sup>-1</sup>	



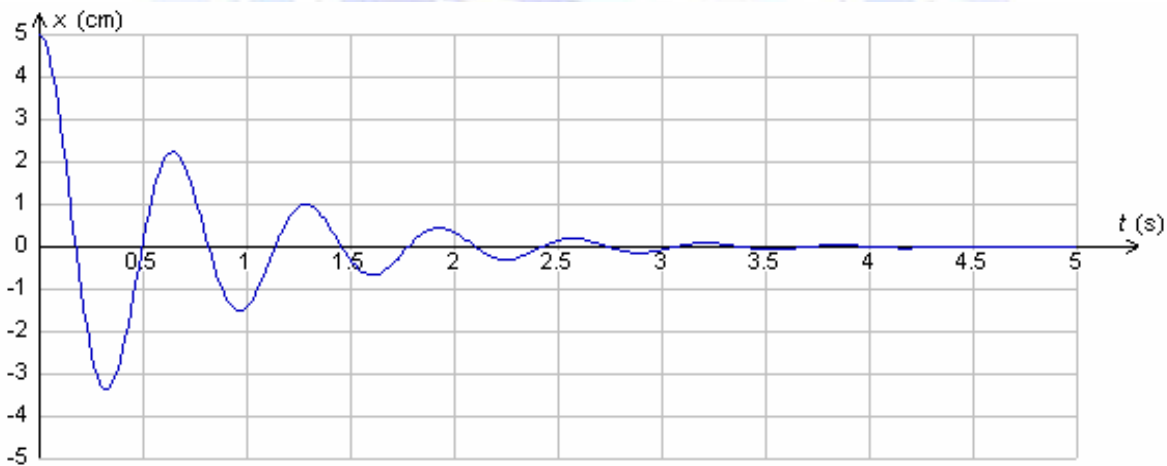
( )

:( )

(Microméga Hatier)

:( .0,5 N.s / m )

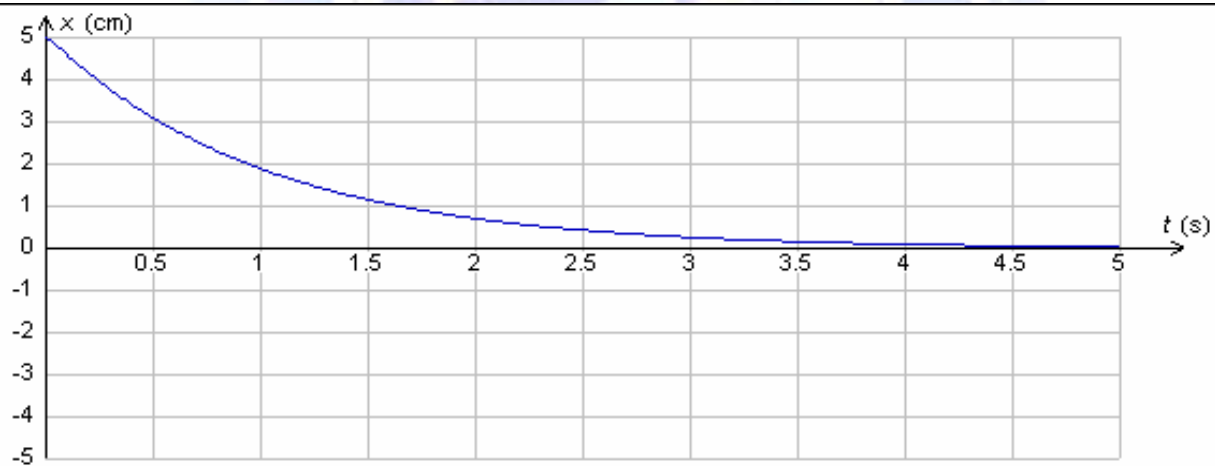
Paramètres	Vecteurs	Affichage	Enregistrement
Position initiale	$x_0$ (-10 à 10) =	<input type="text" value="5"/>	cm
Masse	$m$ (10 à 5 000) =	<input type="text" value="2000"/>	g
Raideur du ressort	$k$ (1 à 1 000) =	<input type="text" value="19,5"/>	N · m <sup>-1</sup>
Coefficient de frottement	$h$ (0 à 1 000) =	<input type="text" value="1,5"/>	N · s · m <sup>-1</sup>



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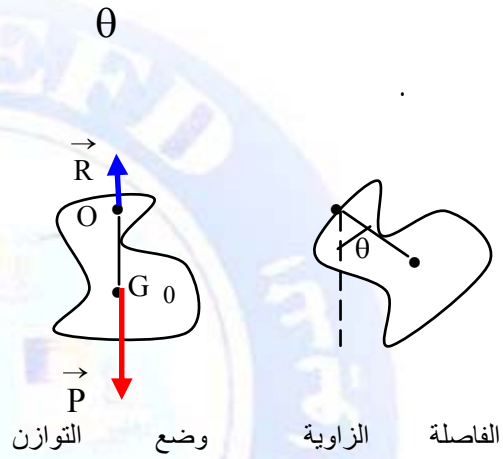
: (200 N.s / m )

Paramètres	Vecteurs	Affichage	Enregistrement
Position initiale	$x_0$ (-10 à 10) =	<input type="text" value="5"/>	cm
Masse	$m$ (10 à 5 000) =	<input type="text" value="2000"/>	g
Raideur du ressort	$k$ (1 à 1 000) =	<input type="text" value="19,5"/>	N · m <sup>-1</sup>
Coefficient de frottement	$h$ (0 à 1 000) =	<input type="text" value="200"/>	N · s · m <sup>-1</sup>



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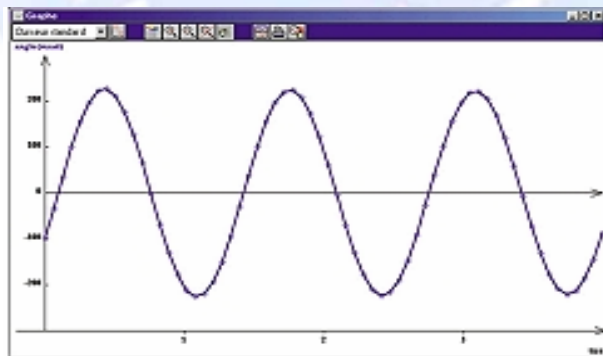
-1-3 :



$\theta$

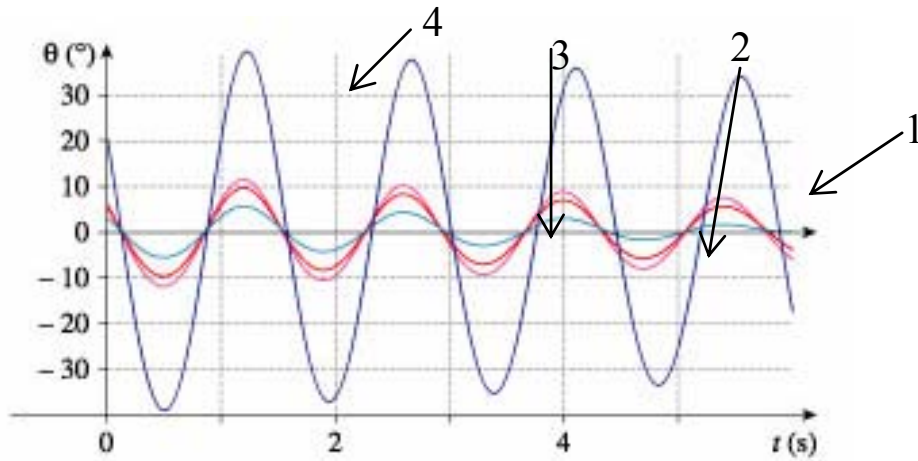
$\cdot T$

$$= \theta_m - \theta_m + \theta_m$$



-2-3

:



3 2 1  
 $\theta < 10^\circ$   
4

:

$\theta < 10^\circ$

( )

-3-3

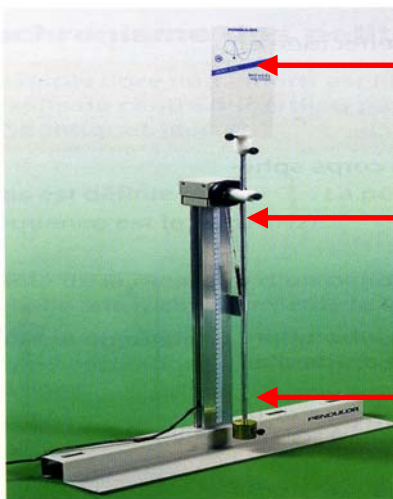
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(Δ)

G

$t_{i+1} - t_i = \tau$   $\theta(t_i)$

5°

t = 0

(.)

$\theta(t)$

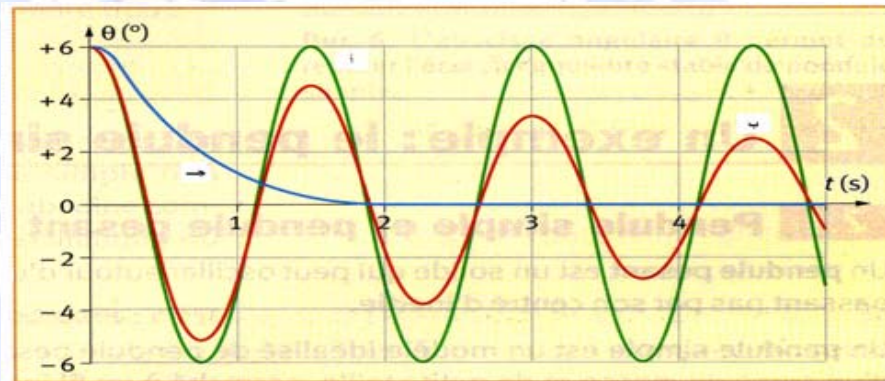
(.)

A<sub>1</sub>

(.)

A<sub>1</sub>

A<sub>2</sub>



-1

T<sub>0</sub> ( )

-2

A<sub>1</sub>

-3

-4

T

-5

( )

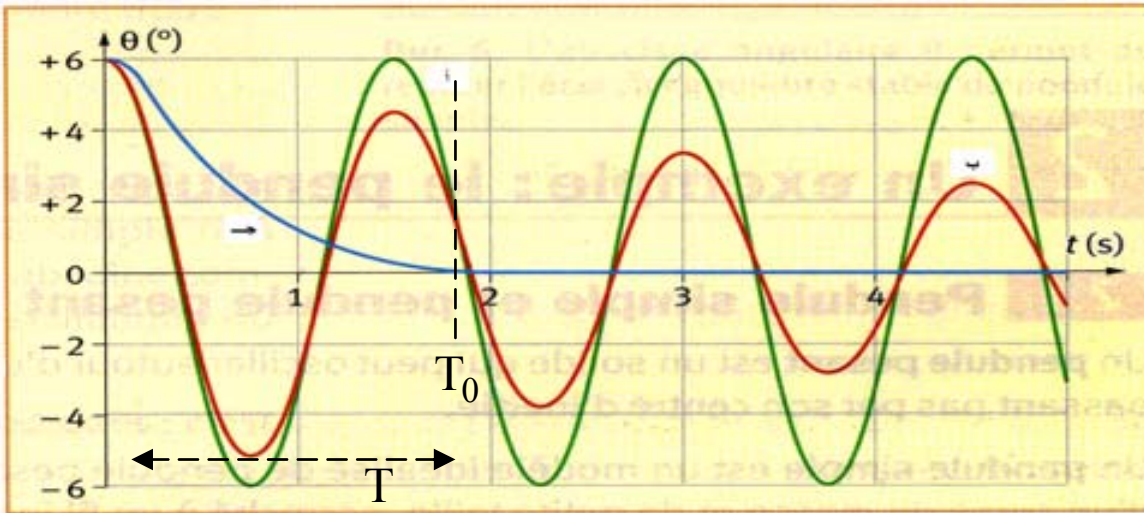
-6



:

( ) -1

: -2



.  $T_0 = 1,5 \text{ s}$  ( )

: -3

( )

: -4

T

: -5

.  $T = 1,5 \text{ s}$  ( )

.  $T = T_0$

-6