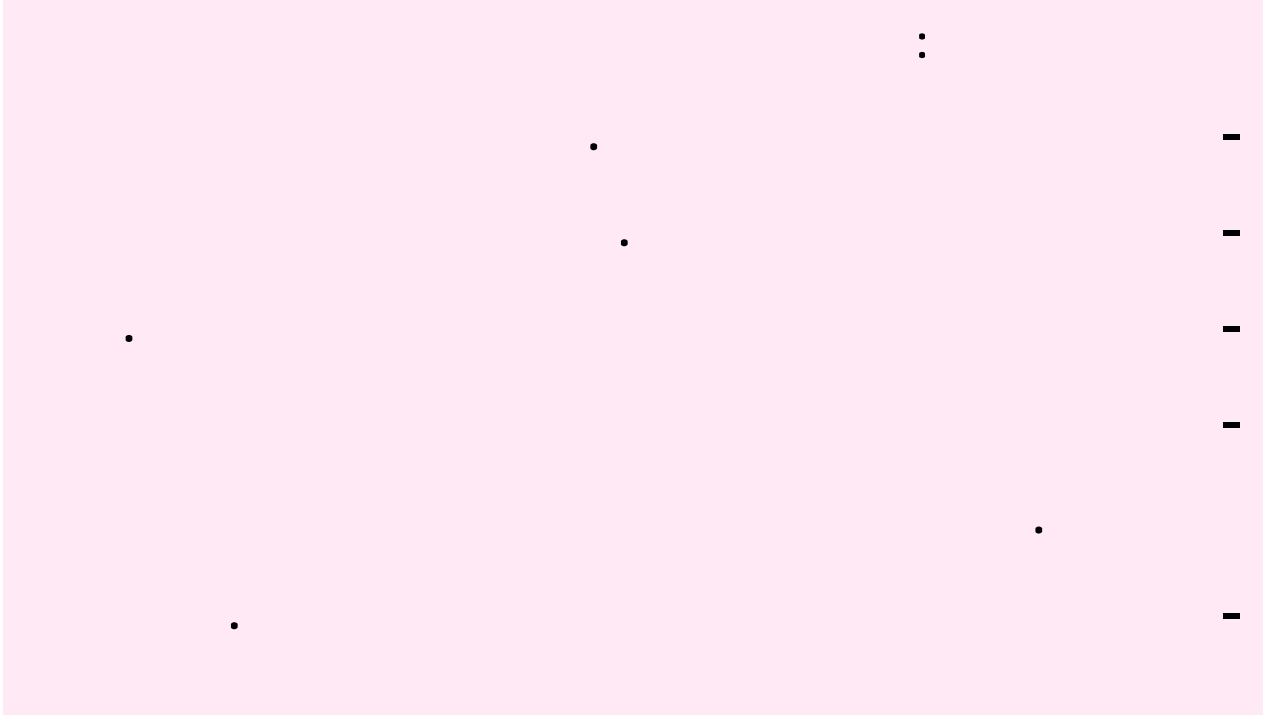


# \* Z



**.Z**

$$L = a \cdot 10^k$$

$$H = a \cdot 10^m$$

$$L < H$$

$$H = 380 \quad L = 240$$

$$PGCD(H; L) = a$$

$$a = PGCD(380; 240)$$

$$380 = 2^2 \times 5 \times 19 \quad 240 = 2^4 \times 3 \times 5$$

$$PGCD(380; 240) = 2^2 \times 5 = 20$$

$$a = 20$$



$$x = \alpha q$$

$$\alpha / x \quad \alpha \quad x$$

:

$$2007 = 3 \times 669 \quad : \quad 2007 \quad 3$$

$$1954 = (-2)(-977) \quad : \quad 1954 \quad (2-)$$

1

0

$$1962 = 5q \quad : \quad q \quad 1962 \quad 5$$

:1

$\gamma, \beta, \alpha$

$$\gamma \quad \alpha \quad \gamma \quad \beta \quad \beta \quad \alpha$$

:

$$(1) \dots \beta = q_1 \alpha \quad : \quad q_1 \quad : \quad \beta \quad \alpha$$

$$q_2 \quad : \quad \gamma \quad \beta$$

$$(2) \dots \gamma = q_2 \beta \quad :$$

$$q \quad \gamma = (q_2 q_1) \alpha \quad : \quad (2) \quad (1)$$

$$\gamma \quad \alpha \quad : \quad (q = q_2 \cdot q_1) \gamma = q \alpha \quad :$$

:2

$$\beta x \quad \alpha \quad x \quad \alpha \quad . \quad x, \beta, \alpha$$

:

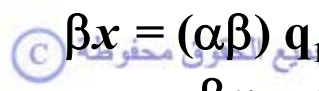
$$q_1 \quad : \quad x \quad \alpha$$

$$. x = \alpha q_1 \quad :$$

<http://www.onefd.edu.tz>

$$\beta x = (\alpha \beta) q_1 \quad :$$

$$\beta x \quad \alpha \quad : \quad \beta x = \alpha q$$



: 3

$$\beta x = \alpha \beta q \quad \text{and} \quad x = \alpha q$$

:

$$\beta x = (\alpha \beta) q \quad \text{and} \quad x = \alpha q$$

$$\beta x = (\alpha \beta) q$$

$$\beta x = \alpha \beta q$$

: 4

$$y = x + b + a + \alpha$$

$$b + a = \alpha$$

$$ax = by$$

:

$$by = ax + \alpha \quad (1) \quad b + a = \alpha$$

$$q_2 \text{ و } q_1$$

$$ax + by = \alpha q_1 + \alpha q_2 \quad \text{and} \quad by = \alpha q_2 \quad \text{and} \quad ax = \alpha q_1$$

$$q_1 + q_2 = q \quad \text{and} \quad ax + by = \alpha(q_1 + q_2)$$

$$ax + by = \alpha q \quad \text{and} \quad ax + by = \alpha q$$

: 5

$$b + a$$

$$a = -b \quad \text{and} \quad a = b \quad \text{and} \quad a = b \quad \text{and} \quad b = a$$

:

$$b = a \cdot k_1 \quad \text{and} \quad k_1 = \frac{b}{a}$$

$$b = a \cdot k_2 \quad \text{and} \quad k_2 = \frac{a}{b}$$

$$b = b \cdot k_1 k_2$$

$$k_2 = 1 \quad \text{and} \quad k_1 = 1 \quad \text{and} \quad k_1 \cdot k_2 = 1$$

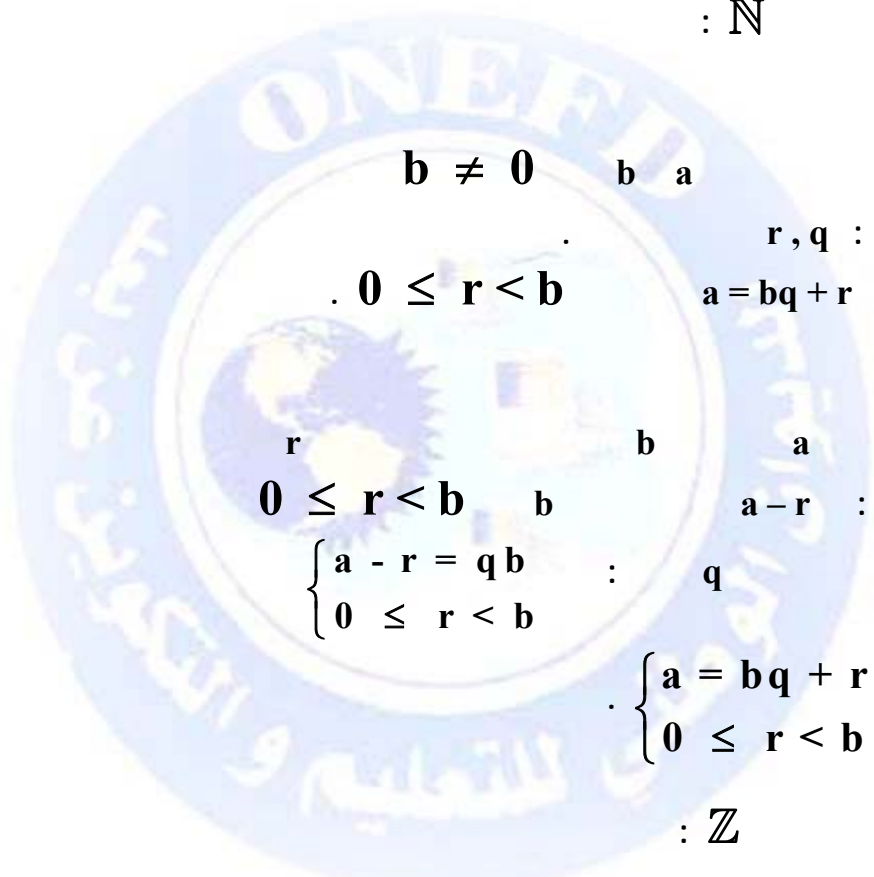
$$k_2 = -1 \quad \text{and} \quad k_1 = -1$$

$$a = -b \quad \text{and} \quad a = b$$



$$\begin{aligned} & : \\ & \cdot -1 \quad 1 \quad \mathbb{Z} \quad 1 \\ & \cdot -1 \quad 1 \quad \mathbb{Z} \quad -1 \end{aligned}$$

**$\mathbb{Z}$**



$:\mathbb{N}$  -

$$\begin{aligned} & \mathbf{b \neq 0} \quad \mathbf{b \mid a} \\ & \cdot \quad \mathbf{r, q : (q ; r)} \\ & \cdot \quad \mathbf{0 \leq r < b} \quad \mathbf{a = bq + r} \end{aligned}$$

$$\begin{aligned} & \mathbf{r} \quad \mathbf{b} \quad \mathbf{a} \\ & \mathbf{0 \leq r < b} \quad \mathbf{b} \quad \mathbf{a - r} : \\ & \left\{ \begin{array}{l} \mathbf{a - r = qb} \\ \mathbf{0 \leq r < b} \end{array} \right. : \quad \mathbf{q} \end{aligned}$$

$$\cdot \left\{ \begin{array}{l} \mathbf{a = bq + r} \\ \mathbf{0 \leq r < b} \end{array} \right. :$$

$:\mathbb{Z}$  -

$$\begin{aligned} & \mathbf{b \neq 0} \quad \mathbf{a \mid b} \\ & \left\{ \begin{array}{l} \mathbf{a = bq + r} \\ \mathbf{0 \leq r < b} \end{array} \right. : \quad \mathbf{(r \mid a) (q ; r)} \end{aligned}$$

$\cdot ( \quad ) \quad \mathbf{a}$  •

$(q' ; r')$

$\cdot \mathbf{-a > 0 : a < 0}$  •

$$\begin{cases} -a = bq' + r' \\ 0 \leq r' < b \end{cases} :$$

$$a = b(-q') : r' = 0 :$$

$$a = b(-q') - r' : r' \neq 0$$

$$a = b(-q' - 1) + b - r' : a = b(-q') - b + b - r' :$$

$$-b < -r' < 0 : 0 < r' < b :$$

$$-q' - 1 = q \quad b - r' = r : 0 < b - r' < b :$$

$$\begin{cases} a = bq + r \\ 0 \leq r < b \end{cases} :$$

: 1

$$2007 = 208 \times 9 + 135 : b = 208 , a = 2007$$

$$q = 9 \text{ و } r = 135 :$$

: 2

$$-1830 = 54(-34) + 6 : b = 54 , a = -1830$$

$$r = 6 \text{ و } q = -34 :$$

: 3

$$b = 166 , a = -1518$$

$$-1815 = 166(-10) + 144$$

$$r = 144 \text{ و } q = -10 :$$



: -  
:

.  $D_a$  a

$$D_0 = \mathbb{N} \quad D_1 = \{1\} :$$

$$D_{28} = \{1 ; 2 ; 4 ; 7 ; 14 ; 28\}$$

:1  
 $D_a \bullet$

. a  $1 \in D_a$

:2  
 $D_a$

.  $a \neq 0$  a

-  
:

. b a

: b a

$$D_{a,b} \quad D_a \cap D_b$$

:

$$D_{30,25} = \{1, 5\}$$

$$a \neq 0 : D_{a,0} = D_a :$$

-

:

$$D_{a,b} \quad b a$$

$$D_{a,b}$$

$$. PGCD(a ; b) : b a$$

$$\text{PGCD}(a; b) = 1$$

$$\text{PGCD}(28, 9) = 1$$

$$a > b$$

$$\begin{cases} a = bq + r_0 \\ 0 \leq r_0 < b \end{cases}$$

$$r_0 = a - bq$$

$$b = a - r_0$$

$$r_0 = b - r_0$$

$$r_0 = bq$$

$$r_0 = b$$

$$b = a - r_0$$

$$D_{a,b} = D_{b,r_0}$$

$$D_{a,b} = D_{b,0} = D_b \quad : \quad r_0 = 0$$

$$b = a$$

$$0 \leq r_1 < r_0 \quad b = r_0 q_1 + r_1 \quad : \quad r_0 \neq 0$$

$$D_{b,r_0} = D_{r_0,r_1}$$

$$D_{a,b} = D_{r_0,r_1}$$

$$D_{a,b} = D_{r_0} \quad : \quad r_1 = 0$$

$$: \quad r_1 \neq 0$$

$$a = bq + r_0 \quad , \quad r_0 < b \quad D_{a,b} = D_{b,r_0}$$

$$b = r_0 \cdot q_1 + r_1 \quad , \quad r_1 < r_0 \quad D_{b,r_0} = D_{r_0,r_1}$$

$$r_0 = r_1 \cdot q_2 + r_2 \quad , \quad r_2 < r_1 \quad D_{r_0,r_1} = D_{r_1,r_2}$$

⋮



$$r_p = r_{p+1} \cdot q_{p+2} + r_{p+2}, \quad r_{p+2} < r_{p+1} \quad D_{r_p, r_{p+1}} = D_{r_{p+1}, r_{p+2}}$$

$$r_{p+2}, \dots, r_1, r_0$$

$$b > r_0 > r_1 > \dots > r_{p+2} > \dots :$$

b

r

$$D_{a,b} = D_{b,r_0} = \dots = D_{r,0} = D_r :$$

. r

b a

:

$D_r$

r

$$: \quad \text{PGCD}(a ; b) = r :$$

:1

: 2

:1

$$D_{1260,440} \quad \text{PGCD}(1260 ; 440)$$

:

$$1260 = 440 \times 2 + 360$$

$$440 = 360 \times 1 + 60$$

$$360 = 60 \times 6 + 20$$

$$60 = 20 \times 3 + 0$$

$$\text{PGCD}(1260 ; 440) = 20 :$$

$$D_{1260,440} = D_{20} = \{1, 2, 4, 5, 10, 20\} :$$

:2

1954 , 2008

:

**PGCD (2008 ; 1954)**

$$2008 = 1954 \times 1 + 54$$

$$1954 = 54 + 36 + 10$$

$$54 = 10 \times 5 + 4$$

$$10 = 4 \times 2 + 2$$

$$4 = 2 \times 2 + 0$$

$$\text{PGCD (2008 ; 1954)} = 2 :$$

$$D_{2008,1954} = D_2 = \{1, 2\} :$$

:3

783 18641

:

**: PGCD (18641 , 783)**

$$18641 = 783 \times 23 + 632$$

$$783 = 632 \times 1 + 151$$

$$632 = 151 \times 4 + 28$$

$$151 = 28 \times 5 + 11$$

$$28 = 11 \times 2 + 6$$

$$11 = 6 \times 1 + 5$$

$$6 = 5 \times 1 + 1$$

$$5 = 1 \times 5 + 0$$

$$\text{PGCD (18641 , 783)} = 1 :$$

783 18641

:3

$\lambda, b, a$

$$\text{PGCD}(\lambda a ; \lambda b) = \lambda \text{PGCD}(a ; b) :$$

$$\text{PGCD}(a ; b) = r_n$$

$$a = bq + r_0, r_0 < b$$

$$b = r_0q_1 + r_1, r_1 < r_0$$

$$r_0 = r_1q_2 + r_2, r_2 < r_1$$

⋮

$$r_{n-2} = r_{n-1}q_n + r_n, r_n < r_{n-1}$$

$$r_{n-1} = r_n \cdot q_{n+1} + 0$$

$$\lambda a = (\lambda b) q_0 + \lambda r_0, \lambda r_0 < \lambda b$$

$$\lambda b = (\lambda r_0) q_1 + \lambda r_1, \lambda r_1 < \lambda r_0$$

$$\lambda r_0 = (\lambda r_1) q_2 + \lambda r_2, \lambda r_2 < \lambda r_1$$

⋮

$$\lambda r_{n-2} = (\lambda r_{n-1}) q_n + \lambda r_n, \lambda r_n < \lambda r_{n-1}$$

$$\lambda r_{n-1} = (\lambda r_n) q_{n+1} + 0$$

$$\text{PGCD}(\lambda a ; \lambda b) = \lambda r_n$$

$$\text{PGCD}(\lambda a ; \lambda b) = \lambda \text{PGCD}(a ; b)$$

: 4

$c$

$B, a$

$$\text{PGCD}(a ; b) = c \cdot \text{PGCD}\left(\frac{a}{c} ; \frac{b}{c}\right)$$

:

$$: \quad b = c \cdot b' \quad \text{و} \quad a = c \cdot a' :$$

$$\text{PGCD}(a ; b) = \text{PGCD}(ca' ; cb')$$

$$= c \times \text{PGCD}(a' ; b')$$

$$= c \times \text{PGCD}\left(\frac{a}{c} ; \frac{b}{c}\right)$$

: 5

• c, b, a

$$\begin{cases} a = C \cdot a' \\ b = C \cdot b' \\ \text{PGCD}(a'; b') = 1 \end{cases} \quad \text{PGCD}(a; b) = C$$

:

$$\begin{cases} a = c \cdot a' \\ b = c \cdot b' \end{cases} : \quad \text{PGCD}(a; b) = c$$

$$\text{PGCD}(a; b) = \text{PGCD}(c \cdot a'; c \cdot b') :$$

$$c = c \cdot \text{PGCD}(a'; b') :$$

$$\text{PGCD}(a'; b') = \frac{c}{c} = 1 :$$

$$\begin{cases} a = c \cdot a' \\ b = c \cdot b' \\ \text{PGCD}(a'; b') = 1 \end{cases} :$$

:

$$\text{PGCD}(a; b) = 12 \quad \begin{matrix} b & a \\ a + b = 120 \end{matrix}$$

:

$$\begin{cases} a = 12a' \\ b = 12b' \\ \text{PGCD}(a'; b') = 1 \end{cases} : \quad \text{PGCD}(a; b) = 12$$

$$12a' + 12b' = 120 : \quad a + b = 120$$

$$a' + b' = 10 : \quad 12(a' + b') = 120 :$$

$$b = 108 \quad a = 12 : \quad b' = 9 \quad a' = 1 \bullet$$

$$b = 84 \quad a = 36 : \quad b' = 7 \quad a' = 3 \bullet$$

. 84 36 108 12

1

$$xy - 2x - 4y + 8 = 10 : y \ x$$

2

$$x^2 - y^2 = 40 : y \ x$$

3

$$q \quad 37 \quad a$$

$$\cdot 11q + 7$$

4

$$b \neq 0$$

$$b \ a$$

$$. r \quad q \quad b^3 \quad a \quad . b \quad a$$

5

$$b \ a \ . \quad n$$

$$. a \quad n \quad q$$

$$. ab \quad n \quad q' : b \ q$$

q'

6

110

7

$$a \times b = 2500 : \quad b \ a$$

$$PGCD (a ; b) = 10$$

$$. b \ a$$

8

$$a^2 - b^2 = 5760 : \quad b \ a$$

$$a < b \quad PGCD (a ; b) = 12$$

$b \ a$

9

$b \ a$   
:

$$PGCD(a ; b) = 10 \quad 2a^2 + 3b^2 = 3500$$

$b \ a$

10



$b \ a \ 7080 \text{ cm}$

. 60

$b \ a$

-

1

$y \ x$

$$xy - 2x - 4y + 8 = 10 :$$

$$x(y - 2) - 4(y - 2) = 10 :$$

$$(y - 2)(x - 4) = 10 :$$

$\cdot y = 12$	$x = 5$	$y - 2 = 10$	$x - 4 = 1$	$\bullet$
$\cdot y = 7$	$x = 6$	$y - 2 = 5$	$x - 4 = 2$	$\bullet$
$\cdot y = 3$	$x = 14$	$y - 2 = 1$	$x - 4 = 10$	$\bullet$
$\cdot y = 4$	$x = 9$	$y - 2 = 2$	$x - 4 = 5$	$\bullet$
$\cdot y = -8$	$x = 3$	$y - 2 = -10$	$x - 4 = -1$	$\bullet$
$\cdot y = -3$	$x = 2$	$y - 2 = -5$	$x - 4 = -2$	$\bullet$
$\cdot y = 1$	$x = -6$	$y - 2 = -1$	$x - 4 = -10$	$\bullet$
$\cdot y = 0$	$x = -1$	$y - 2 = -2$	$x - 4 = -5$	$\bullet$

2

$$x^2 - y^2 = 40 : \quad y \ x$$

$$x - y < x + y : \quad (x - y)(x + y) = 40 :$$

$\cdot 2x = 41 :$	$x + y = 40$	$x - y = 1$	(1)
$\cdot 2x = 22 :$	$x + y = 20$	$x - y = 2$	(2)
	$\cdot y = 9$	$x = 11$	:
$2x = 14 :$	$x + y = 10$	$x - y = 4$	(3)
	$\cdot y = 3$	$x = 7$	:
( ) $2x = 13 :$	$x + y = 8$	$x - y = 5$	(4)

$$\begin{aligned}
 & : a \\
 & \begin{cases} a = 37q + 11q + 7 \\ 0 \leq 11q + 7 < 37 \end{cases} : \\
 -7 \leq 11q < 30 & : \quad 0 \leq 11q + 7 < 37 : \\
 \frac{-7}{11} \leq q < \frac{30}{11} & : \\
 -0,63 \leq q < 2,72 & : \\
 q \in \{0 ; 1 ; 2\} & : \\
 & : \\
 & a = 7 : q = 0 \\
 & a = 55 \quad a = 37 + 11 + 7 : q = 1 \\
 a = 103 & : \quad a = 37(2) + 11 \times 2 + 7 : q = 2 \\
 & . 103, 55, 7 : a
 \end{aligned}$$

$$\begin{aligned}
 & . b \quad a \\
 & \begin{cases} a = b(b^2q) + r \\ 0 \leq r < b^3 \end{cases} : \quad \begin{cases} a = b^3q + r \\ 0 \leq r < b^3 \end{cases} : \\
 r = bq_1 + r_1 & : \quad b \quad r \\
 & 0 \leq r_1 < b \\
 & \begin{cases} a = b(b^2q) + bq_1 + r_1 \\ 0 \leq r_1 < b \end{cases} : \\
 & \begin{cases} a = b(b^2q + q_1) + r_1 \\ 0 \leq r_1 < b \end{cases} : \\
 & . \quad b^2q + q_1 \quad b \quad a \quad r_1
 \end{aligned}$$



5

$$\begin{cases} q = bq' + r' \\ 0 \leq r' \leq b - 1 \end{cases} \quad \begin{cases} n = aq + r \\ 0 \leq r \leq a - 1 \end{cases} :$$

$$n = a(bq' + r') + r :$$

$$n = (ab)q' + ar' + r$$

$$0 \leq ar' \leq a(b - 1) : 0 \leq r' \leq b - 1$$

$$0 \leq ar' + r \leq a(b - 1) + a - 1 :$$

$$0 \leq ar' + r \leq ab - a :$$

$$0 \leq ar' + r < ab :$$

$$ab - a < ab :$$

$$\begin{cases} n = (ab) \cdot q' + (ar' + r) \\ 0 \leq ar' + r < ab \end{cases} :$$

$$\cdot q' \quad ab \quad n \quad :$$

6

$$: \quad q \quad r \quad . \quad a$$

$$\begin{cases} a = 110 \times q + q^2 \\ q^2 < 110 \end{cases} :$$

$$q < 10,48 : q^2 < 110$$

$$q \in \{0 ; 1 ; \dots ; 10\} :$$

$$a = q(110 + q)$$

:

$$a = 111 : q = 1 \quad (2) \quad a = 0 : q = 0 \quad (1)$$

$$a = 339 : q = 3 \quad (4) \quad a = 224 : q = 2 \quad (3)$$

$$a = 575 : q = 5 \quad (6) \quad a = 456 : q = 4 \quad (5)$$

$$a = 819 : q = 7 \quad (8) \quad a = 696 : q = 6 \quad (7)$$

$$a = 1071 : q = 9 \quad (10) \quad a = 944 : q = 8 \quad (9)$$

$$a = 1200 : q = 10 \quad (11)$$

7

b a

$$b = 10b' \quad a = 10a'$$

b' و a'

$$10a' \cdot 10b' = 2500 : \quad a \times b = 2500$$

$$a' \cdot b' = 25$$

$$b = 250 \quad a = 10 \quad b' = 25 \quad a' = 1 \quad \bullet$$

$$b = 10 \quad a = 250 \quad b' = 1 \quad a' = 25 \quad \bullet$$

8

: b a

$$b' و a' \quad b = 12 b' \quad a = 12 a' :$$

$$a' < b' : \quad a < b :$$

$$a^2 - b^2 = 5760 :$$

$$(12a')^2 - (12b')^2 = 5760 :$$

$$(12)^2 [a'^2 - b'^2] = 5760 :$$

$$144 (a'^2 - b'^2) = 5760 :$$

$$a'^2 - b'^2 = 40 :$$

$$(a' - b')(a' + b') = 40 :$$

$$2a' = 41 \quad a' + b' = 40 \quad a' - b' = 1 : \quad \bullet$$

$$\bullet \text{ لما : } a' - b' = 2 \text{ و } a' + b' = 20 \text{ بالجمع نجد } 2a' = 22$$

$$\bullet \text{ ومنه : } a' = 11 \text{ بجمع } a' = 11 \text{ و } b' = 9 \text{ . } \text{http://www.onefd.edu.dz}$$

$$\text{إذن : } a = 12 \times 11 = 132 \text{ , } b = 12 \times 9 = 108$$

$$2a' = 14 : \quad a' + b' = 8 \quad a' - b' = 4 \quad \bullet$$

$$b' = 3 \text{ و } a' = 7 : \quad \bullet$$

$$b = 12 \times 3 = 36 , \quad a = 12 \times 7 = 84 :$$

$$2a' = 13 : \quad a' + b' = 8 \text{ و } a' - b' = 5 : \quad \bullet$$

9

: b a

$$b' \text{ و } a' \quad b = 10b' \quad a = 10a' :$$

$$2(10a')^2 + 3(10b')^2 = 3500 :$$

$$10^2 [2a'^2 + 3b'^2] = 3500 :$$

$$2a'^2 + 3b'^2 = 35 :$$

$$3b'^2 = 35 - 2a'^2 :$$

$$a'^2 \leq \frac{35}{2} : \quad 35 - 2a'^2 \geq 0 :$$

$$a' \leq 4 : \quad a' \leq \sqrt{\frac{35}{2}}$$

$$b'^2 = 11 : \quad 3b'^2 = 33 : a' = 1 \bullet$$

$$b' = 3 : \quad b'^2 = 9 : \quad 3b'^2 = 27 : a' = 2 \bullet$$

$$b = 30 \quad a = 20 :$$

$$3b'^2 = 17 : a' = 3 \bullet$$

$$b'^2 = 1 : \quad 3b'^2 = 3 : a' = 4 \bullet$$

$$b' = 1 :$$

$$b = 10 \quad a = 40 :$$

3540

$$. a + b = 3540$$

$$b' \quad a'$$

. 60

$$\begin{cases} a = 60 a' \\ b = 60 b' \end{cases} :$$

$$60 a' + 60 b' = 3540 : \quad b' \quad a'$$

$$a' + b' = 59 : \quad 60(a' + b') = 3540 :$$

$$a' = b' + 1 :$$

$$b' + 1 + b' = 59 :$$

$$b' = 29 \quad 2b' = 58 :$$

$$a' = 30 :$$

$$b = 60 \times 29 \quad , \quad a = 60 \times 30 :$$

$$. b = 1760 \quad , \quad a = 1800 :$$